Quantum nonlocality without hidden variables: An algorithmic approach

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Is quantum mechanics (QM) local or nonlocal? Different formulations/interpretations (FI) of QM, with or without hidden variables, suggest different answers. Different FI's can be viewed as different algorithms, which leads us to propose an algorithmic definition of locality according to which a theory is local if and only if there exists at least one FI in which all irreducible elements of that FI are local. The fact that no such FI of QM is known strongly supports quantum nonlocality.

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Is quantum mechanics (QM) local or nonlocal? The no-hidden-variable theorems, such as those of Bell [1], Greenberger, Horne, and Zeilinger [2], and Hardy [3], prove that no local hidden variable theory can reproduce the predictions of QM. This means that if hidden variables (i.e., objective elements of reality existing even without measurements) exist, then they cannot be local. Some take this as an indication that hidden variables cannot exist, while others take this as an indication that QM is nonlocal. Thus, to definitely answer the question whether QM is local or not, we need a definition of locality that does not rest on the notion of (hypothetical) hidden variables.

One such widely used definition of locality is *signal* locality. According to this definition, QM is local if it does not allow to send a signal (information) that propagates faster than light. Clearly, according to this definition, QM is local. Nevertheless, although it is widely appreciated that QM obeys signal locality, there is still no consensus regarding the question whether QM is local or not. In particular, nonlocal hidden-variable interpretations of QM, such as the Bohmian interpretation [4], also obey signal locality, but this does not stop the opponents of hidden-variable interpretations to criticize such interpretations for not being local. Thus, signal locality is not a completely satisfying definition of locality either. To achieve a consensus regarding the question of quantum locality, we obviously need a different, more general, definition of locality itself.

Since the origin of all quantum nonlocalities can be reduced to the entanglement between different particles, it is sometimes argued that there is nothing really nonlocal about QM because particles need to perform a local interaction in order to come into an entangled state. However, even if it is true in practical experiments, it is not true in principle. Even the free Schrödinger equation of two noninteracting particles contains entangled solutions, while no theoretical principle of QM forbids such solutions.

Last but not least, adherents of different interpretations of QM typically have different opinions on quantum locality. For example, adherents of hidden-variable interpretations, conceiving that elements of objective reality must exist even if we do not measure them, typically find QM intrinsically nonlocal. On the other hand, adherents of some interpretations that explicitly deny the existence of a unique objective reality, such as the many-world interpretation [5, 6, 7] and the relational interpretation [8, 9], argue that these interpretations save locality of QM. Still, by definition, all these interpretations have the same observable predictions; they are all *observationally equivalent*. Thus, to achieve a consensus regarding the question of quantum locality, it is very important to have a definition that does not depend on the interpretation.

Now we have rather strong constraints on a satisfying definition of locality. On the one hand, it should not be a purely observational definition (e.g., like signal locality), because the adherents of hidden-variable interpretations will complain that observational locality does not exclude the existence of nonlocal hidden variables. On the other hand, it should be an interpretation-independent definition, so that, in particular, it does not depend on whether the existence of hidden variables is assumed or not. How to fulfill both of these two somewhat opposite constraints? As the formalism of QM is the aspect of QM that is usually not regarded as controversial, our idea is to use a *formal-theoretical* definition of locality.

For simplicity, we study two-particle systems with particle positions $\mathbf{x}$ and $\mathbf{y}$. Formal-theoretical elements of the form $f(\mathbf{x})$ or $f(\mathbf{y})$ (where f may be a function, an operator, or whatever) that depend only on $\mathbf{x}$ or only on $\mathbf{y}$ are said to be local. By contrast, a formal-theoretical element of the form $f(\mathbf{x}, \mathbf{y})$ depending on both $\mathbf{x}$ and $\mathbf{y}$ is said to be nonlocal. Clearly, the existence of nonlocal elements is a sign of possible nonlocality of the theory. Indeed, the source of all nonlocal properties of QM can be traced back to the formal nonlocality of wave functions $\psi(\mathbf{x}, \mathbf{y})$, living in the configuration space, rather than in the ordinary 3-space. Nevertheless, the existence of nonlocal elements in a theory is certainly not yet the proof of nonlocality of the theory. To see this, it is instructive to discuss some simple examples.

Consider a system of two *classical* particles. It is certainly possible to build a classical statistical ensemble in which the positions of the particles are correlated, such that the probability distribution $P(\mathbf{x},\mathbf{y})$ depends on both $\mathbf{x}$ and $\mathbf{y}$. Clearly, $P(\mathbf{x},\mathbf{y})$ is a nonlocal element of the classical theory. But does it mean that classical mechanics is nonlocal? Certainly not. To see that this nonlocal element does not necessarily imply nonlocality of the theory, one may recall that classical mechanics can be formulated in terms of hidden variables, which are particle positions existing even when they are not measured. (Note, however, that all predictions of classical mechanics can be reproduced even without such hidden variables [10].) For example, if particles have time-dependent trajectories $\mathbf{x} = \mathbf{v}t$, $\mathbf{y} = \mathbf{b}+\mathbf{v}t$, then $P(\mathbf{x},\mathbf{y}) = \delta^3(\mathbf{x}-\mathbf{y}+\mathbf{b})$. Thus, although $P(\mathbf{x},\mathbf{y})$ is nonlocal, this nonlocal correlation between $\mathbf{x}$ and $\mathbf{y}$ can be recovered from hidden variables $\mathbf{x}$ and $\mathbf{y}$ that represent local elements of the theory.

The example above is certainly not sufficiently general to prove locality of classical mechanics. For a general proof, one needs a *general formulation* of two-particle classical mechanics. One such formulation sufficiently general for our purposes is the Hamilton-Jacobi formulation, where the central formal-theoretical element is the nonlocal function $S(\mathbf{x},\mathbf{y},t)$ satisfying the Hamilton-Jacobi equation

$$\frac{(\nabla_x S)^2}{2m_1} + \frac{(\nabla_y S)^2}{2m_2} + V_1(\mathbf{x}) + V_2(\mathbf{y}) = -\partial_t S. \quad (1)$$

Clearly, the Hamilton-Jacobi formulation is not local. However, when the Hamilton-Jacobi equation is combined with another fundamental element of the Hamilton-Jacobi formulation

$$m_1\dot{\mathbf{x}}(t) = \nabla_x S, \quad m_2\dot{\mathbf{y}}(t) = \nabla_y S, \quad (2)$$

then various different but observationally equivalent formulations of classical mechanics can be obtained. Examples are the Hamiltonian and the Lagrangian formulations, which are also nonlocal, because the fundamental elements (Hamiltonians and Lagrangians) of these formulations are nonlocal. Nevertheless, one of the formulations, the Newton formulation consisting of the two equations

$$m_1\ddot{\mathbf{x}}(t) = -\nabla_x V_1(\mathbf{x}), \quad m_2\ddot{\mathbf{y}}(t) = -\nabla_y V_2(\mathbf{y}), \quad (3)$$

does not contain any nonlocal elements. Thus, *the fact that there exists at least one formulation of classical mechanics that does not contain nonlocal elements proves that classical mechanics is local!*

We also emphasize that the requirement of locality does not exclude interactions between particles. The locality of interactions is the most explicitly expressed in field theories, where a finite number of derivatives of a field $\phi(\mathbf{x})$ is regarded as a local object.

Now let us turn back to QM. The Hamilton-Jacobi formulation of classical mechanics above is very similar to the Schrödinger formulation of QM. The Schrödinger formulation describing the nonlocal wave function $\psi(\mathbf{x},\mathbf{y},t) = R(\mathbf{x},\mathbf{y},t)\exp[iS(\mathbf{x},\mathbf{y},t)/\hbar]$ is indeed nonlocal. By analogy with classical mechanics, one might hope that one could combine the Schrödinger equation with Eq. (2) to obtain an observationally equivalent formulation that, analogously to the Newton formulation of classical mechanics, does not contain nonlocal elements. Indeed, the resulting formulation really turns out to be observationally equivalent to the Schrödinger formulation. Nevertheless, it does not eliminate all nonlocal elements from the theory. Instead of the local Newton equations (3) one obtains nonlocal quantum analogues [4]

$$\begin{aligned} m_1\ddot{\mathbf{x}}(t) &= -\nabla_x[V_1(\mathbf{x}) + Q(\mathbf{x},\mathbf{y},t)], \\ m_2\ddot{\mathbf{y}}(t) &= -\nabla_y[V_2(\mathbf{y}) + Q(\mathbf{x},\mathbf{y},t)], \end{aligned} \quad (4)$$

where $Q(\mathbf{x},\mathbf{y},t)$ is the nonlocal quantum potential [4].

The formulation of QM containing Eqs. (4) is known as the Bohmian interpretation [4]. (We believe that, in general, it is not possible to clearly distinguish between the notions of "formulations" and "interpretations", as long as both different formulations and different interpretations are observationally equivalent. For example, the Bohmian interpretation can be viewed as a practically useful formulation [11], while the Schrödinger formulation may be viewed as an interpretation by someone who thinks of a wave function as a physical object evolving with time. This is why we use the expressions "formulation" and "interpretation" interchangeably.) It is in fact the Bohmian formulation that inspired Bell to find his famous theorem on quantum nonlocality and hidden variables. Nevertheless, this does not prove nonlocality of QM itself, as different formulations of QM without hidden variables are also possible.

Abstractly, a physical theory can be viewed as an equivalence class of *all* observationally equivalent formulations/interpretations (FI) of that theory. If some property of a theory is not manifest in one FI, it may be manifest in another. An FI of a theory is nothing but a set of irreducible elements (axioms, definitions, as well as primitive "common-sense" objects that are not explicitly defined), which are mutually independent (no axiom or definition can be deduced from other axioms and definitions) and from which all other properties of the theory can be deduced. Even more abstractly, any such FI can be viewed as an *algorithm* for determinination of reducible elements of the theory, such as specific observable predictions of physical theories. In the case of QM, these observable predictions are probabilities, such as the joint probability $P(\mathbf{x},\mathbf{y})$ in a specific physical configuration. The probabilities themselves as general elements not attributed to a specific physical configuration may or may not be irreducible elements of an FI. When they are not irreducible elements in an FI, we say that the probabilities are emergent in that FI. For example, the probabilities are emergent in the deterministic formulation of classical mechanics, in some versions of the Bohmian deterministic formulation of QM [12, 13], as well as in some versions of the many-world interpretation [14, 15, 16] of QM. However, in most FI's of QM the probabilities are fundamental, i.e., correspond to some irreducible elements of the FI.

From the observations above we see that locality of an FI is manifest if and only if all irreducible elements of that FI are local. Since the theory is the equivalence class of all observationally equivalent FI's of that theory, it motivates us to define locality of the theory as follows: *A theory is local if and only if there exists an FI of the theory in which all irreducible elements are local.* Since any FI can be viewed more abstractly as an algorithm, we refer to this definition of locality as *algorithmic locality.* Indeed, the definition above can be rephrased more concisely as follows: *A theory is local if and only if it can be represented by a local algorithm.*

The main motivation for such an abstract definition of locality is to provide a resolution of the old controversy regarding the question whether QM is local or nonlocal. As a tentative answer to this question, we propose the following *algorithmic nonlocality conjecture*: *No local algorithm can reproduce the probabilistic predictions of QM.*

Note that this conjecture sounds very similar to the already proven hidden-variable nonlocality theorem saying that *no local hidden variables can reproduce the probabilistic predictions of QM.* Indeed, all abstract irreducible elements of an algorithm that are not directly observable may be viewed as "hidden variables". However, the notion of hidden variables actually has a more specific meaning, denoting elements that are *observable*, but defined even when they are not observed. Thus, our algorithmic nonlocality conjecture is much more general (and thus much more difficult to prove) than the hidden-variable nonlocality theorem.

Although we do not (yet) have a proof of the algorithmic nonlocality conjecture, there is a lot of evidence supporting it. In the following we present some of this evidence by supporting it by some specific FI's of QM.

It is often argued that QM is essentially only about (local) information (see, e.g., [17, 18] and references therein). The relational interpretation [8, 9] is also a variant of this idea. If QM is only about local information, then signal locality (referring to the propagation of information) is a good criterion of locality. However, if QM is really only about information, then it is reasonable to expect that there should exist a formulation of QM in which *all* irreducible elements refer to information itself. Such a formulation should *not* contain auxiliary elements (such as wave functions or Hamiltonians) from which information can be deduced, but which do not represent information itself. Unfortunately, no such formulation is known and it does not seem that such a formulation is possible. From this we conclude that, in the algorithmic sense, QM does *not* seem to be only about information. Consequently, signal locality does not support algorithmic locality.

A variant of the information-theoretic interpretation of QM contains a claim that QM is essentially only about correlations [19]. However, we know that quantum correlations may be nonlocal (typically, EPR correlations). Thus, even if it is possible to formulate QM only in terms of correlations (which we doubt), it is almost a tautology that these elements of formulation must be nonlocal, which supports algorithmic nonlocality.

One of the origins of quantum nonlocalities is the concept of wave-function collapse. Some no-hidden-variable interpretations, such as the consistent-histories interpretation [20, 21, 22] and the many-world interpretation [5, 6, 7], completely eliminate wave-function collapse from the fundamental formulation of the theory. Nevertheless, these interpretations do not eliminate all nonlocal elements from the fundamental irreducible formulation. In the consistent-histories interpretation, the probabilities of different histories are calculated with the aid of a nonlocal state described by a density matrix in the Heisenberg picture. The many-world interpretation reformulates a quantum superposition $\psi(\mathbf{x},\mathbf{y}) = \sum_a f_a(\mathbf{x})h_a(\mathbf{y})$ as a collection of different "worlds" $\{f_a(\mathbf{x})h_a(\mathbf{y})\}$. Nevertheless, the single "worlds" $f_a(\mathbf{x})h_a(\mathbf{y})$ are still nonlocal objects. The many-world interpretation cannot be further reformulated in terms of local "worlds" $\{f_a(\mathbf{x}), h_a(\mathbf{y})\}$ only, because the specific pairings in which f_1 is paired with h_1, f_2 with h_2, and so on, are an important part of the formulation that allows the correct prediction of observable joint probabilities.

A local formulation of QM should not contain nonlocal states (represented, e.g., by nonlocal wave functions $\psi(\mathbf{x},\mathbf{y})$ or nonlocal density matrices) as irreducible elements of the formulation. The only such formulation we are aware of is the Nelson interpretation [23, 24], which, indeed, eliminates both local and nonlocal wave functions from the fundamental formulation of the theory. However, it is a hidden-variable interpretation, which, of course, is nonlocal.

From the discussion above, we can conclude that no present formulation of QM obeys algorithmic locality, which strongly supports the algorithmic nonlocality conjecture. Nevertheless, let us assume that this conjecture is wrong, that a local formulation, though yet unknown, still exists. In particular, it would mean that all presently known FI's of QM could be viewed as incomplete, which could also be thought of as an algorithmic version of the old EPR argument [25]. An interesting question is: Would that mean that hidden-variable formulations of QM, being observationally equivalent to this hypothetical local formulation, should be considered local? According to our definition of algorithmic locality, the answer is – yes. In this sense, hidden-variable interpretations of QM could be local. However, we emphasize that this is true only if these hidden-variable formulations really *are* observationally equivalent to the local formulation. In this regard, we note that some variants of the Bohmian interpretation may, under certain circumstances, observationally differ from other formulations. The examples of such circumstances are the early universe [26] and certain relativistic conditions [27].

As a final aside remark, not essential for the main thesis of this work, but interesting for a discussion of quantum nonlocalities without hidden variables in a wider context, we note that some aspects of string theory possess some surprising nonlocal features [28, 29] that have no analogs in the quantum theory of particles or fields. The origin of these nonlocalities turns out to lie in certain *dualities* of string theory. Namely, in two different string theories defined on different background manifolds, some observable properties (e.g., the spectrum of the Hamiltonian) turn out to be completely identical. As recognized and illustrated on the example of T-duality in [30], these nonlocalities arise precisely owing to the (tacit) assumption that hidden variables do *not* exist. Namely, the nonlocalities arise from the identification of theories that coincide in certain observable properties, but such an identification makes sense only if all other unobserved properties are unphysical. On the other hand, the assumption of the existence of hidden variables means that unobserved properties may also be physical, which then removes dualities and corresponding nonlocalities at the fundamental hidden-variable level. Nevertheless, this does not remove nonlocalities completely, as all hidden-variable formulations of QM must be nonlocal. These theoretical observations are also indirectly related to the conjecture that quantum nonlocalities cannot be completely removed by reformulations of the theory.

To summarize, in this paper we have motivated a new definition of locality called algorithmic locality and presented evidence for the validity of the algorithmic nonlocality conjecture, according to which no local algorithm can reproduce the probabilistic predictions of QM. Essentially, this conjecture seems to be valid because any formulation of QM seems to require nonlocal wave functions (or some other nonlocal substitute for them) as one of the fundamental irreducible elements of the formulation.

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